

(a)

The minimum values of X, Y and P are all 0

The maximum value of  $X = 1011101_2 = 64 + 16 + 8 + 4 + 1 = 93$

The maximum value of  $Y = 101101_2 = 32 + 8 + 4 + 1 = 45$

The maximum value of  $P = (93)(45) = 4185$

A 12-bit unsigned number can represent values in the range  $[0, 4095]$ , which is not large enough.

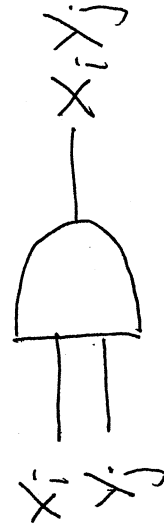
A 13-bit unsigned number can represent values in the range  $[0, 8191]$ , which is large enough.

Therefore, the minimum number of bits needed to represent P is 13.

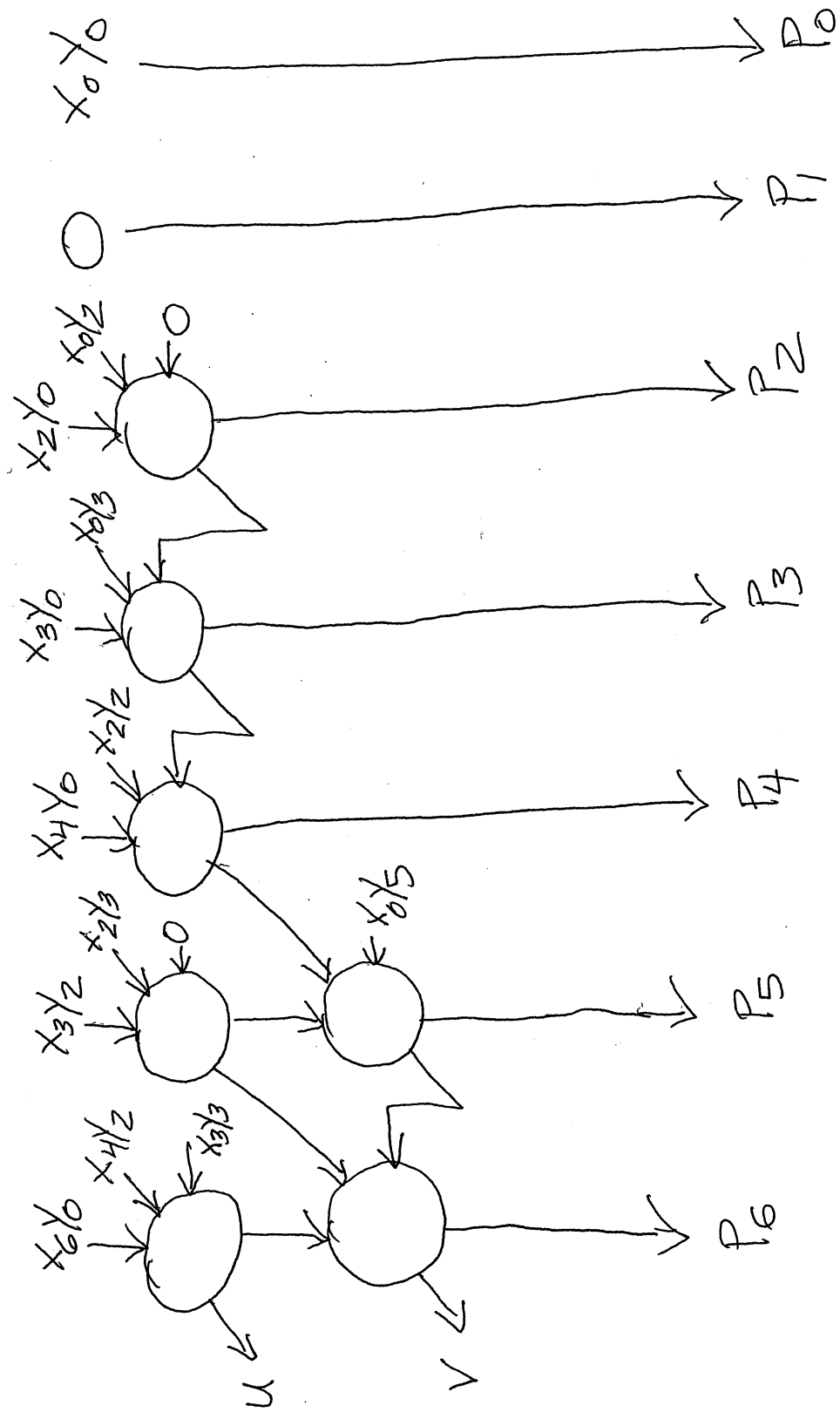
(b)

$x_6$	0	$x_4$	$x_3$	$x_2$	0	$x_0$
	$y_5$	0	$y_3$	$y_2$	0	$y_0$
$x_6 y_0$	0	$x_4 y_0$	$x_3 y_0$	$x_2 y_0$	0	$x_0 y_0$
$x_6 y_2$	0	$x_4 y_2$	$x_3 y_2$	0	$x_0 y_2$	
$x_6 y_3$	0	$x_4 y_3$	$x_3 y_3$	0	$x_0 y_3$	
$x_6 y_5$	0	$x_4 y_5$	$x_3 y_5$	0	$x_0 y_5$	
$P_{12}$	$P_{11}$	$P_{10}$	$P_9$	$P_8$	$P_7$	$P_6$
				$P_5$	$P_4$	$P_3$
					$P_2$	$P_1$
						$P_0$

For each term  $x_i y_j$  in the above, include an AND gate as follows:

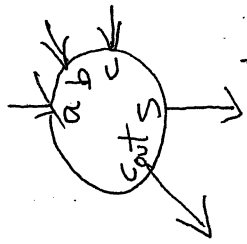


Circuit diagram - Part 1:



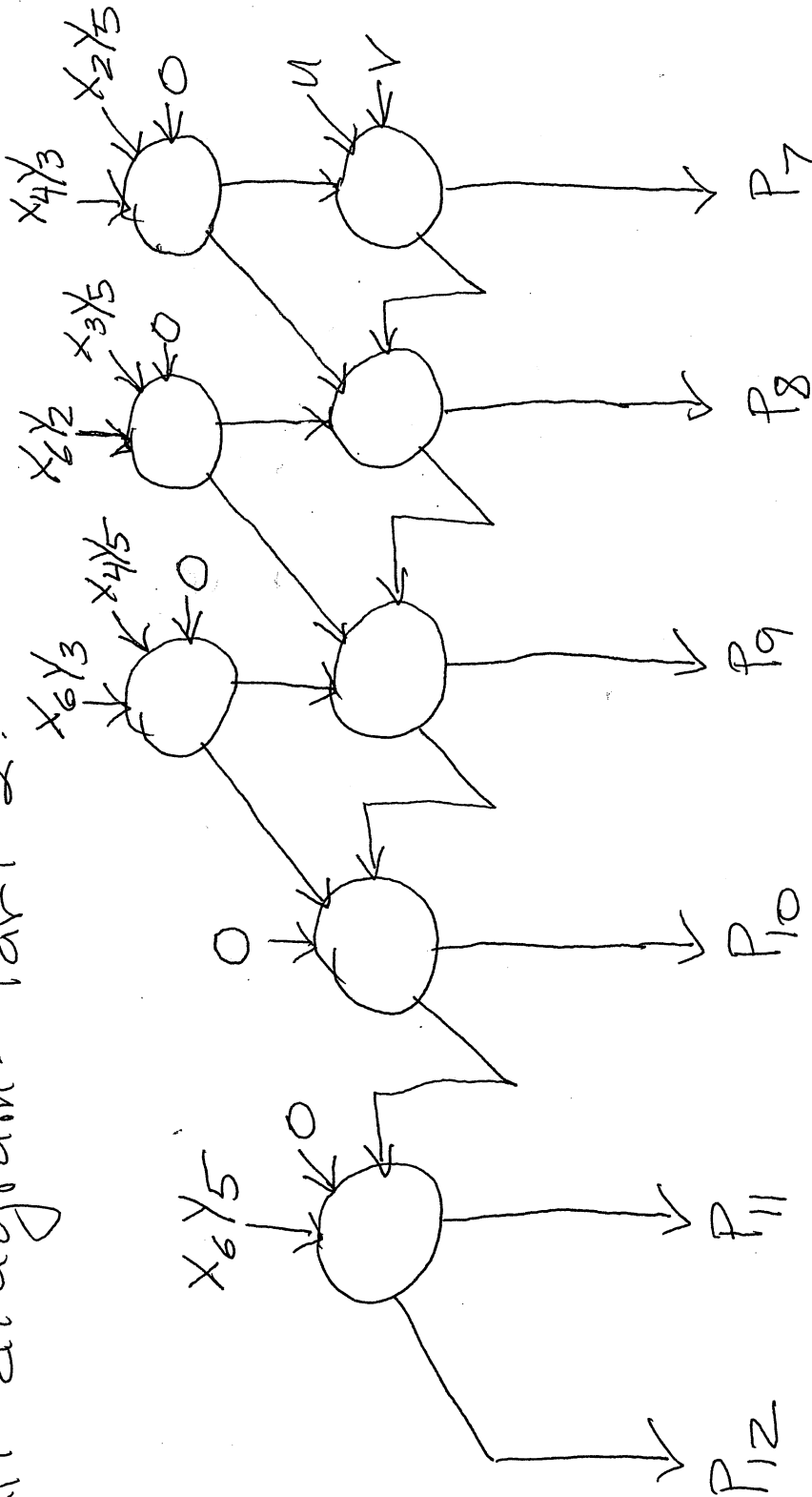
• Notation:

A full adder is:



• Note: Assignment of signals to specific inputs of the full adders within a column is arbitrary.

Circuit diagram - Part 2:



(c)

The minimum value of  $Z = 10111000_2 = -128 + 32 + 16 + 8 = -72$

The maximum value of  $Z = 01111111_2 = 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$

The minimum value of  $W = 0 + (32)(-72) = -2304$

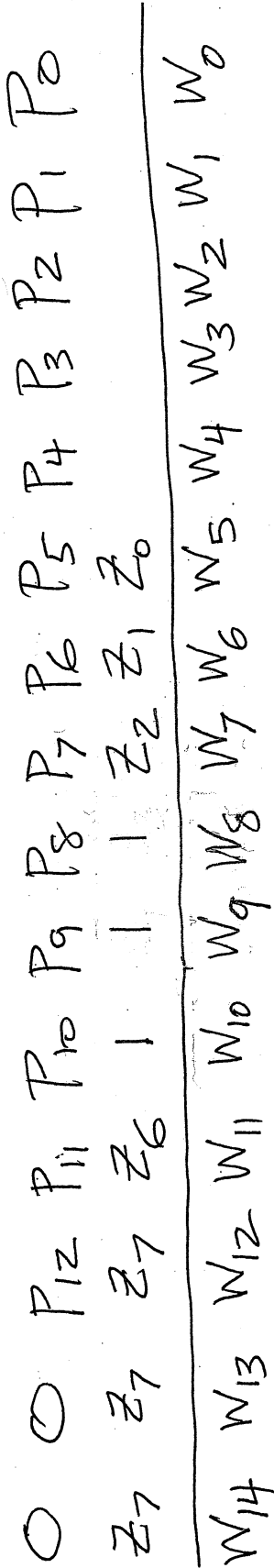
The maximum value of  $W = 4185 + (32)(127) = 8249$

A 14-bit signed number can represent values in the range  $[-8192, 8191]$ , which is not large enough.

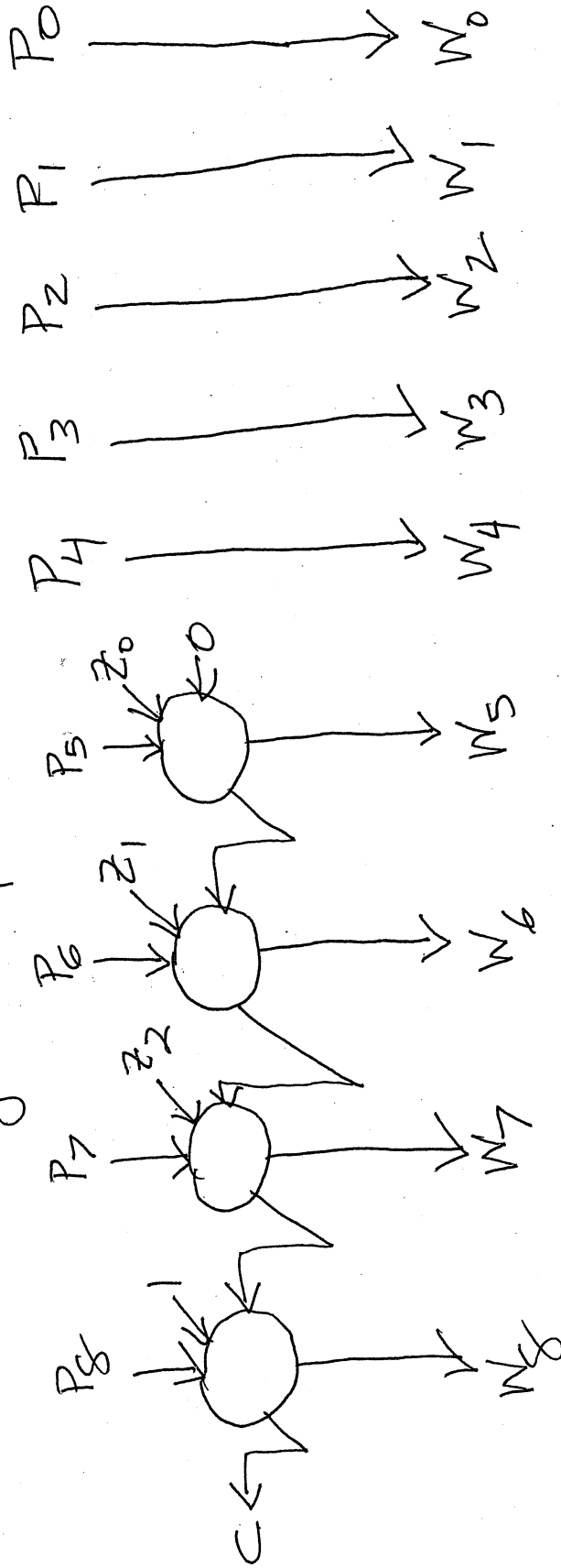
A 15-bit signed number can represent values in the range  $[-16384, 16383]$ , which is large enough.

Therefore, the minimum number of bits need to represent  $W$  without any signed overflow is 15.

(d) sign-extend P and 32Z to 15-bits:



circuit diagram - part 1:



Circuit diagram - Part 2:

